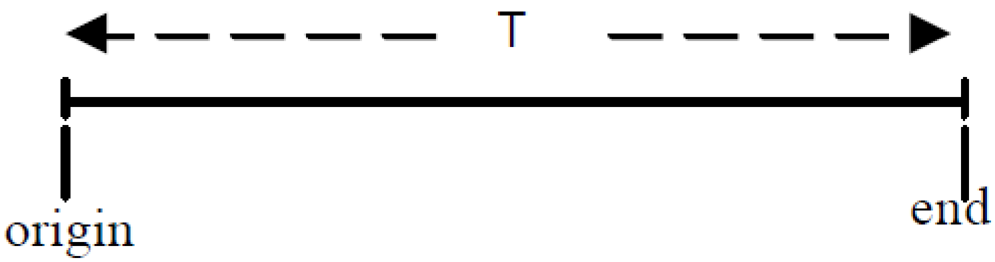
Correlated random effects in survival analysis

Mészáros, G. – Sölkner, J. – Ducrocq, V.

62nd Annual Meeting EAAP, 29. August – 2. September 2011
Session: S.30 Free communications: animal breeding methodology
Abstract no.: 9705
Corresponding author: Gábor Mészáros, Gabor.Meszaros@boku.ac.at

Survival Analysis

- To analyze the time until an event occurs
- Analysis of response time



- Can deal with censored observations and time dependent covariates

Survival Analysis

- General equation:

$$\lambda(t) = \lambda_0 * exp(X(t)_i{'}\beta_i)$$

- $\lambda(t)$ – hazard for an individual
- λ_0 – baseline hazard
- $X(t)_i$ – set of (possibly time dependent) fixed explanatory variables
- β_i – vector of estimates

Survival Analysis – random effect

- Could be extended to incorporate (possibly time dependent) random effects

$$\lambda(t) = \lambda_0 * exp(X(t)_i'\beta_i + Z(t)_i's_i)$$

- Resulting into: $u = (0, \sigma^2)$

- Widely used in genetic analysis for sire or animal models

Survival Analysis– 2 random effects

- Simultaneous estimation of variances for 2 random effects is also possible

$$\lambda(t) = \lambda_0 * exp(X(t)_i'\beta_i + Z_1(t)_i's_{1i} + Z_2(t)_i's_{2i})$$

- $u = \left(0, {\sigma_1}^2\right)$
- $v = \left(0, {\sigma_2}^2\right)$

- *E.g. herd×year and sire effect*

Correlated random effects

- Until now the estimates for the random effects assumed to be independent from each other…
- … although it might not be always the case

- E.g. culling during early vs. late lactations

Correlated random effects

- An extension of the *Survival Kit* was made to incorporate this possibility

$$\lambda(t) = \lambda_0 * exp(X(t)_i'\beta_i + Z_1(t)_i's_{1i} + Z_2(t)_i's_{2i})$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} {\sigma_1}^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & {\sigma_2}^2 \end{pmatrix}$$

Simulation study

- 50 and 100 groups (sires) with 100 individuals (daughters) in each
- Weibull distribution with $\rho=3$
  intercept=0.02
- One fixed effect with 2 levels
- 200 replicates for each alternative

Simulation results

		$\sigma_1{}^2$ (true) = 0.3		$\sigma_2{}^2$ (true) = 0.3		ρ (true) = -0.2
		without ρ	with ρ	without ρ	with ρ	with ρ
50 sires 100 daughters each	$\bar{x}$	0,285	0,285	0,275	0,275	-0,179
	s	0,061	0,060	0,060	0,060	0,150
100 sires 100 daugh. each	$\bar{x}$	0,279	0,282	0,274	0,277	-0,190
	s	0,040	0,041	0,044	0,044	0,107

- $\bar{x}$ and s are the means and standard deviations from the 200 test runs

Simulation results

		$\sigma_1{}^2$ (true) = 0.3		$\sigma_2{}^2$ (true) = 0.3		ρ (true) = -0.6
		without ρ	with ρ	without ρ	with ρ	with ρ
50 sires 100 daughters each	$\bar{x}$	0,268	0,284	0,257	0,274	-0,592
	s	0,058	0,060	0,060	0,062	0,101
100 sires 100 daugh. each	$\bar{x}$	0,262	0,282	0,256	0,277	-0,597
	s	0,039	0,041	0,042	0,044	0,070

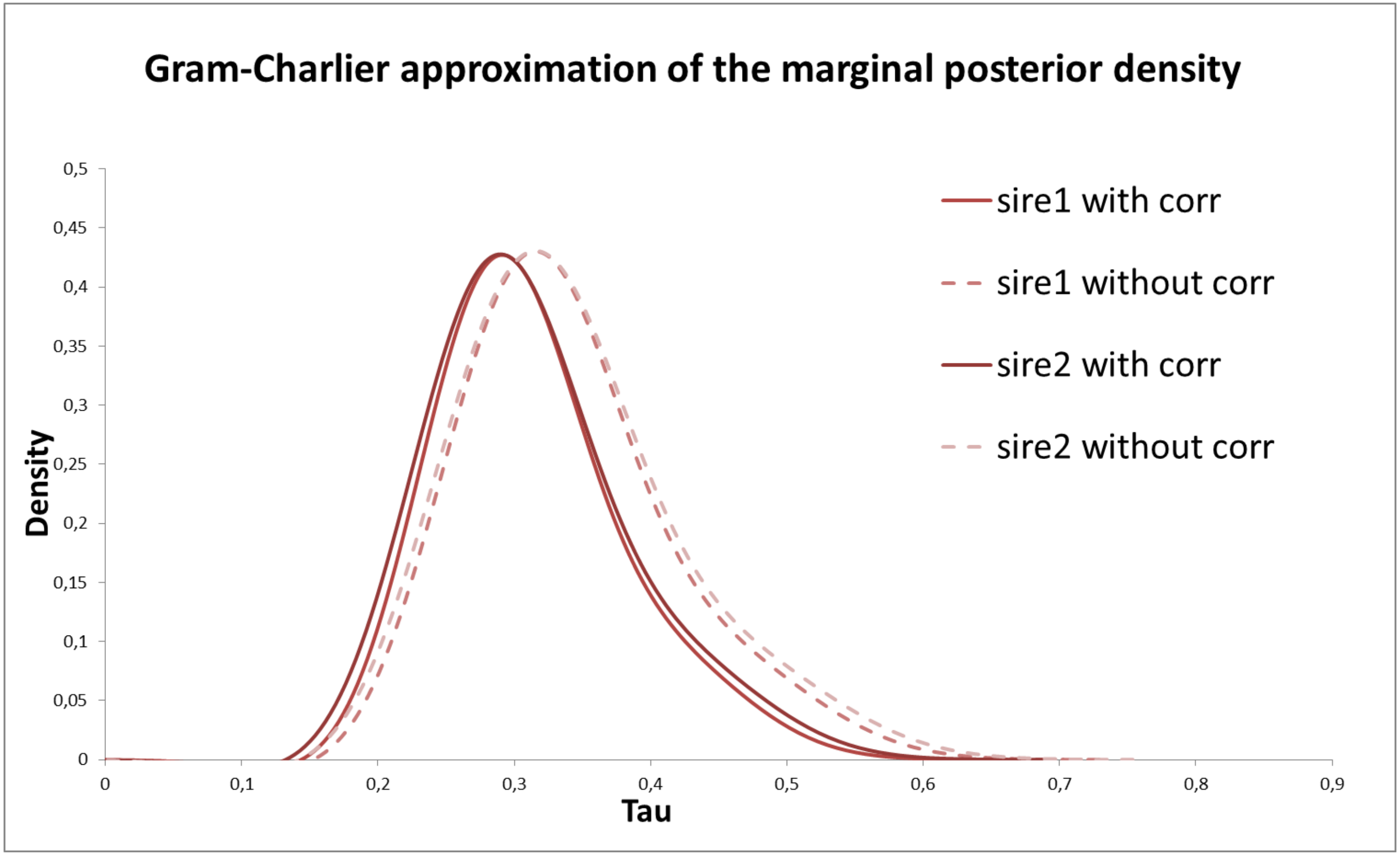
- $\bar{x}$ and s are the means and standard deviations from the 200 test runs

Simulation results

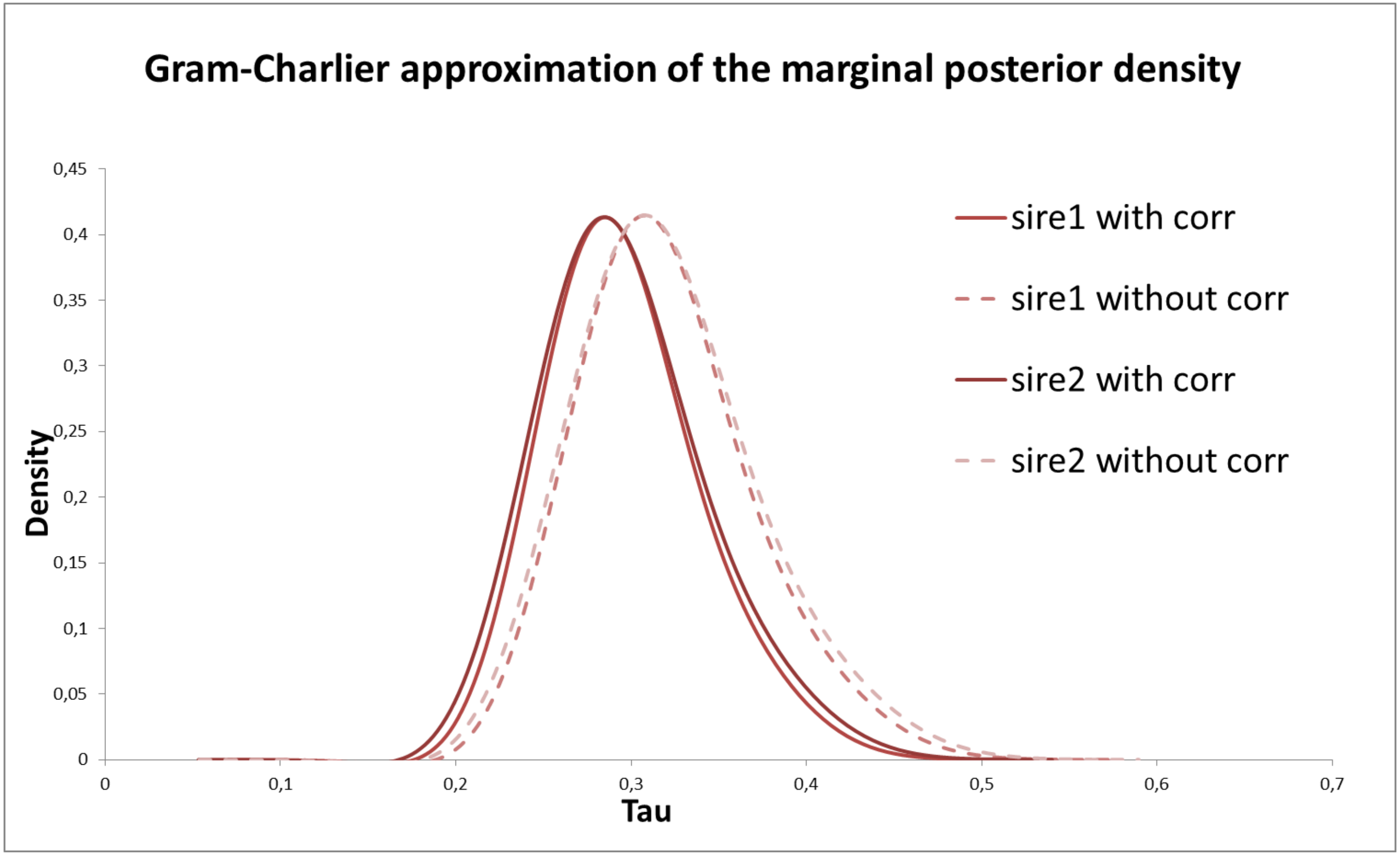
		$\sigma_1{}^2$ (true) = 0.3		$\sigma_2{}^2$ (true) = 0.3		ρ (true) = 0.6
		without ρ	with ρ	without ρ	with ρ	with ρ
50 sires 100 daughters each	$\bar{x}$	0,313	0,285	0,313	0,284	0,622
	s	0,065	0,060	0,073	0,068	0,115
100 sires 100 daugh. each	$\bar{x}$	0,307	0,283	0,307	0,282	0,615
	s	0,043	0,041	0,052	0,049	0,082

- $\bar{x}$ and s are the means and standard deviations from the 200 test runs

Simulation results – corr=0.6
50 sires -100 daughters



Simulation results – corr=0.6 100 sires - 100 daughters



Practical example

- Work in progress
- In cooperation with Birgit Fürst-Waltl
- Influence of the sire on mortality of female calves
- 2 periods:
  - Calf up to 180 days
  - Heifer from 181 days to first calving

Practical example

- "Time dependent sire" random effect along with other fixed effects
- Censoring >96%
- var("early sire effect") = 0.045
  - $h^2 = 0.007$ (cens. not considered: $h^2 = 0.17$)
- var("late sire effect")   = 0.023
  - $h^2 = 0.004$ (cens. not considered: $h^2 = 0.09$)
- correlation = 0.75

Conclusions

- Work in progress
- Little difference in estimates of variances when correlation is small
- Correlations estimated well
- Interesting opportunities of application
  - Direct and maternal genetic effects
  - Time dependent genetic effects
  - …

Thanks for your attention!

Corresponding author: Gábor Mészáros, Gabor.Meszaros@boku.ac.at