# Genomic evaluation for both purebred and crossbred performance

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## Motivation (pigs)

- ► Genomic selection has been implemented for purebreds.
- ▶ Crossbreeding is predominant in the production.
- ▶ Ideally, genomic evaluation should include records on crossbreds, and breeding values for crossbred performance should be presented.

Aim here: develop methods

- ► Two-breed terminal crossbreding
- ► Single-step method (pedigree and genomic)

### Outline

- ► Genomic model for purebreds and crossbreds.
- ▶ Pedigree-based model for purebreds and crossbreds.
- ► Single-step method for purebreds and crossbreds
- ► Future work

### Genomic model for purebreds and crossbreds

Purebreds

$$\begin{aligned} \mathbf{y}_{\mathcal{A}} &= \mu_{\mathcal{A}} \mathbf{1} + \mathbf{m}_{\mathbf{A}} \boldsymbol{\beta}_{\mathcal{A}} + \mathbf{e}_{\mathcal{A}}, \\ \mathbf{y}_{\mathcal{B}} &= \mu_{\mathcal{B}} \mathbf{1} + \mathbf{m}_{\mathbf{B}} \boldsymbol{\beta}_{\mathcal{B}} + \mathbf{e}_{\mathcal{B}}, \end{aligned}$$

where  $\mathbf{m}_{ij}^{\mathcal{A}}=0,1,2$  for genotypes 11, 12, 22 (same for  $\mathbf{m}_{ij}^{\mathcal{B}}).$ 

► Crossbreds (Ibanez et al , 2009):

$$\mathbf{y}_{\mathcal{A}\mathcal{B}} = \mu_{\mathcal{A}\mathcal{B}}\mathbf{1} + \mathbf{q}_{\mathcal{A}}\boldsymbol{\beta}_{\mathcal{A}}^{c} + \mathbf{q}_{\mathcal{B}}\boldsymbol{\beta}_{\mathcal{B}}^{c} + \mathbf{e}_{\mathcal{A}\mathcal{B}}$$

where  $\mathbf{q}_{ij}^{\mathcal{A}}=0,1$  for haplotypes 1, 2 (same for  $\mathbf{q}_{ij}^{\mathcal{B}}$ ).

► Genotypes phased to breed of origin.

### Genomic model for purebreds and crossbreds

$$\begin{split} \mathbf{y}_{\mathcal{A}} &= \mu_{\mathcal{A}} \mathbf{1} + \mathbf{a}_{\mathcal{A}} + \mathbf{e}_{\mathcal{A}}, \\ \mathbf{y}_{\mathcal{B}} &= \mu_{\mathcal{B}} \mathbf{1} + \mathbf{a}_{\mathcal{B}} + \mathbf{e}_{\mathcal{B}}, \\ \mathbf{y}_{\mathcal{A}\mathcal{B}} &= \mu_{\mathcal{A}\mathcal{B}} \mathbf{1} + \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{A}} + \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{B}} + \mathbf{e}_{\mathcal{A}\mathcal{B}} \end{split}$$

- $\blacktriangleright \ \, \text{BVs for purebred performance:} \ \, \textbf{a}_{\mathcal{A}} = \textbf{m}_{\textbf{A}} \boldsymbol{\beta}_{\mathcal{A}} \text{, } \, \textbf{a}_{\mathcal{B}} = \textbf{m}_{\textbf{B}} \boldsymbol{\beta}_{\mathcal{B}}$
- $\qquad \qquad \qquad \textbf{Partial genetic effects:} \ \ \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{A}} = \mathbf{q}_{\mathcal{A}}\boldsymbol{\beta}_{\mathcal{A}}^{c}, \ \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{B}} = \mathbf{q}_{\mathbf{B}}\boldsymbol{\beta}_{\mathcal{B}}^{c}.$
- $\blacktriangleright \ \, \mathsf{BVs} \,\, \mathsf{for} \,\, \mathsf{crossbred} \,\, \mathsf{performance:} \,\, \boldsymbol{c}_{\mathcal{A}} = \boldsymbol{m}_{\mathcal{A}} \boldsymbol{\beta}^{\boldsymbol{c}}_{\mathcal{A}} , \,\, \boldsymbol{c}_{\mathcal{B}} = \boldsymbol{m}_{\mathcal{B}} \boldsymbol{\beta}^{\boldsymbol{c}}_{\mathcal{B}}$
- ▶ Equiv. model: 2 Genomic breed specific "partial" relationship matrices,  $\mathbf{G}^{(\mathcal{A})}$ ,  $\mathbf{G}^{(\mathcal{B})}$ .

## Genomic breed A partial relationship matrix

$$\mathbf{G}^{(\mathcal{A})} = \left[ egin{array}{ccc} \mathbf{G}_{\mathcal{A},\mathcal{A}}^{(\mathcal{A})} & \mathbf{G}_{\mathcal{A},\mathcal{AB}}^{(\mathcal{A})} \ \mathbf{G}_{\mathcal{AB},\mathcal{A}}^{(\mathcal{A})} & \mathbf{G}_{\mathcal{AB},\mathcal{AB}}^{(\mathcal{A})} \end{array} 
ight]$$

$$\begin{split} \mathbf{G}_{\mathcal{A},\mathcal{A}}^{(\mathcal{A})} &= \frac{(\mathbf{m}^{\mathcal{A}} - 2\bar{\boldsymbol{\rho}}^{\mathcal{A}}\mathbf{1}^{\mathrm{T}})(\mathbf{m}^{\mathcal{A}} - 2\bar{\boldsymbol{\rho}}^{\mathcal{A}}\mathbf{1}^{\mathrm{T}})^{\mathrm{T}}}{s^{(\mathcal{A})}}, \\ \mathbf{G}_{\mathcal{A},\mathcal{A}\mathcal{B}}^{(\mathcal{A})} &= \frac{(\mathbf{m}^{\mathcal{A}} - 2\bar{\boldsymbol{\rho}}^{\mathcal{A}} - \mathbf{1}^{\mathrm{T}})(\mathbf{q}^{\mathcal{A}} - \bar{\boldsymbol{\rho}}^{\mathcal{A}}\mathbf{1}^{\mathrm{T}})^{\mathrm{T}}}{s^{(\mathcal{A})}}, \\ \mathbf{G}_{\mathcal{A}\mathcal{B},\mathcal{A}\mathcal{B}}^{(\mathcal{A})} &= \frac{(\mathbf{q}^{\mathcal{A}} - \bar{\boldsymbol{\rho}}^{\mathcal{A}}\mathbf{1}^{\mathrm{T}})(\mathbf{q}^{\mathcal{A}} - \bar{\boldsymbol{\rho}}^{\mathcal{A}}\mathbf{1}^{\mathrm{T}})^{\mathrm{T}}}{s^{(\mathcal{A})}}. \end{split}$$

- ▶ For breed  $\mathcal{A}$ :  $\mathbf{m}_{ij}^{\mathcal{A}} = 0, 1, 2$  for genotypes 11, 12, 22
- ▶ For crossbreds:  $\mathbf{q}_{ij}^{\mathcal{A}} = 0, 1$  for haplotypes 1, 2.
- lacktriangle Allele frequency:  $ar{
  ho}_j$  .

## Pedigree-based model

(Wei and van der Werf, 1994)

$$\begin{split} \mathbf{y}_{\mathcal{A}} &= \mu_{\mathcal{A}} \mathbf{1} + \mathbf{Z}_{\mathcal{A}} \mathbf{a}_{\mathcal{A}} + \mathbf{e}_{\mathcal{A}}, \\ \mathbf{y}_{\mathcal{B}} &= \mu_{\mathcal{B}} \mathbf{1} + \mathbf{Z}_{\mathcal{B}} \mathbf{a}_{\mathcal{B}} + \mathbf{e}_{\mathcal{B}}, \\ \mathbf{y}_{\mathcal{A}\mathcal{B}} &= \mu_{\mathcal{A}\mathcal{B}} \mathbf{1} + \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{A}} + \mathbf{c}_{\mathcal{A}\mathcal{B}}^{\mathcal{B}} + \mathbf{e}_{\mathcal{A}\mathcal{B}}, \end{split}$$

where

- ▶ BVs for purebred performance:  $\mathbf{a}_{\mathcal{A}}$ ,  $\mathbf{a}_{\mathcal{B}}$ .
- ightharpoonup Breed of origin specific genetic effects:  $\mathbf{c}_{\mathcal{AB}}^{\mathcal{A}},\,\mathbf{c}_{\mathcal{AB}}^{\mathcal{B}},$
- ▶ BVs for crossbred performance:  $\mathbf{c}_{\mathcal{A}}$ ,  $\mathbf{c}_{\mathcal{B}}$ .

## Pedigree-based model

- ▶ BVs for crossbred performance  $\mathbf{c}_{\mathcal{A}}$  are correlated with BVs for purebred performance  $\mathbf{a}_{\mathcal{A}}$  (genetic correlation) and with partial genetic effects  $\mathbf{c}_{\mathcal{A}\mathcal{B}}^{(\mathcal{A})}$  (partial pedigree relationships).
- ► Supports a breeding goal with both purebred and crossbred performance.
- ▶ Allows different genetic variances in two breeds.
- ▶  $\mathbf{c}_{\mathcal{A}}$ ,  $\mathbf{c}_{\mathcal{B}}$  captures dominance effects, different genetic background,  $\mathsf{G} \times \mathsf{E}$

#### Genetic covariances

▶ Breed A:

$$\mathsf{Var} \left[ \begin{array}{c} \mathbf{a}_{\mathcal{A}}^{(\mathcal{A})} \\ \star \\ \mathbf{c}_{\mathcal{A}} \\ \mathbf{c}_{\mathcal{A}\mathcal{B}}^{(\mathcal{A})} \end{array} \right] = \mathbf{\Sigma}_{\mathcal{B}}^{(\mathcal{A})} \bigotimes \mathbf{A}^{(\mathcal{A})}$$

where  $\mathbf{A}^{(\mathcal{A})}$  is partial relationship matrix for breed  $\mathcal{A}$  (Garcia-Cortes and Toro, 2006).

- $ightharpoonup \Sigma_g^{(\mathcal{A})}$ : 2 × 2 genetic variance-covariance matrix
- ightharpoonup Breed  $\mathcal B$  is similar.
- ightharpoonup Breed  ${\mathcal A}$  and breed  ${\mathcal B}$  specific genetic effects are independent.

# Partial relationship matrix $\mathbf{A}^{(\mathcal{A})}$

Recursions (Garcia-Cortes, 1996):

$$\mathbf{A}_{i,i'}^{(\mathcal{A})} = (\mathbf{A}_{f(i),i'}^{(\mathcal{A})} + \mathbf{A}_{m(i),i'}^{(\mathcal{A})})/2$$

$$\mathbf{A}_{i,i}^{(\mathcal{A})} = f_i^{\mathcal{A}} + (\mathbf{A}_{f(i),i'}^{(\mathcal{A})} + \mathbf{A}_{m(i),i'}^{(\mathcal{A})})/2$$

where  $f_i^{\mathcal{A}} = 1, 0, 1/2$  for breed  $\mathcal{A}$ , breed  $\mathcal{B}$ , and crossbred  $\mathcal{AB}$ .

- lacktriangle Ignore breed  ${\cal B}$  animals in relationship matrix.
- ▶ Sparse inverse can be computed directly.
- ▶ Submatrix can be computed using Colleau (2002) algorithm.

## Combined relationship matrix

(pedigree and genomic)

► Combined partial relationship matrix:

$$(\mathbf{H}^{(\mathcal{A})})^{-1} = \begin{bmatrix} (\mathbf{G}^{(\mathcal{A})})^{-1} - (\mathbf{A}_{11}^{(\mathcal{A})})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + (\mathbf{A}^{(\mathcal{A})})^{-1}, \quad (1)$$

- $ightharpoonup {f G}_a^{({\cal A})}$ : genomic partial relationship matrix
- ▶ Sparse inverse  $(\mathbf{A}^{(\mathcal{A})})^{-1}$  computed directly.
- $ightharpoonup oldsymbol{\mathsf{A}}_{11}^{(\mathcal{A})}$  computed using Colleau algorithm

### Single-step method

- ► Partial relationship matrices based on pedigree are replaced by combined partial relationship matrices.
- ▶ Can be fitted using standard software for multi-trait genetic evaluation using  $(\mathbf{H}^{(\mathcal{A})})^{-1}$  and  $(\mathbf{H}^{(\mathcal{B})})^{-1}$  as input.
- ► Compatibility of partial **G** and **A**: Linear adjustment of partial **G** towards partial **A**.
- ► Assumption: marker-data for crossbred animals can be phased to breed of origin.
- ► Method is available!

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#### Future work

- ► PhD student Tao Xiang will use the model on maternal trait in Landrace (L), Yorkshire (Y) and crossbred (LY).
- Questions of interest to investigate:
  - ► Genetic correlation between purebred and crossbred performance.
  - Accuracy of breeding values for crossbred performance.
- ▶ Extension to three-breed terminal crossbreeding is possible.