# Predicting the accuracy of multi-population genomic prediction

#### **Yvonne Wientjes**

\_\_\_

2<sup>nd</sup> September, 2015

Piter Bijma, Roel Veerkamp, Mario Calus







## Acknowledgements







## Designing breeding programs

## Maximize response to selection



## Designing breeding programs

## Maximize response to selection



Accuracy for different designs



## Accuracy of genomic selection

Size of reference population

Accuracy of genomic prediction

## Accuracy of genomic selection

Size of reference population

Accuracy of genomic prediction

## **Combine populations:**

- Countries
- Lines

- Breeds
- Bulls/Cows



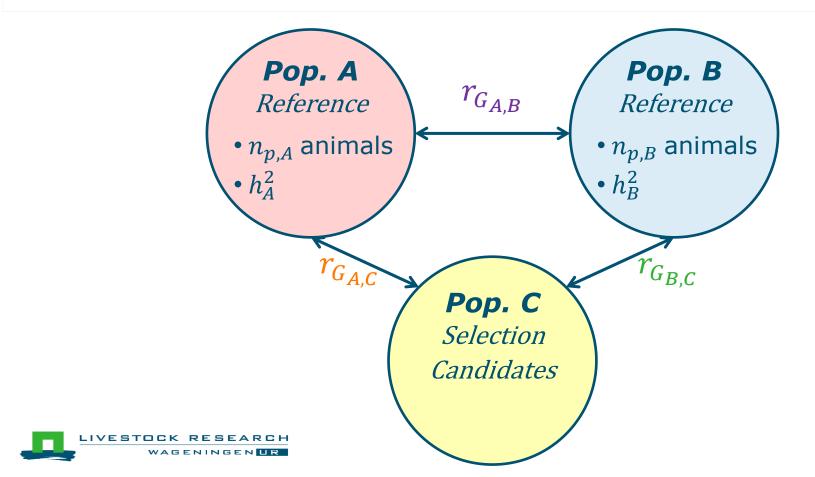
## Aim of this study

### **Derive** and **validate**:

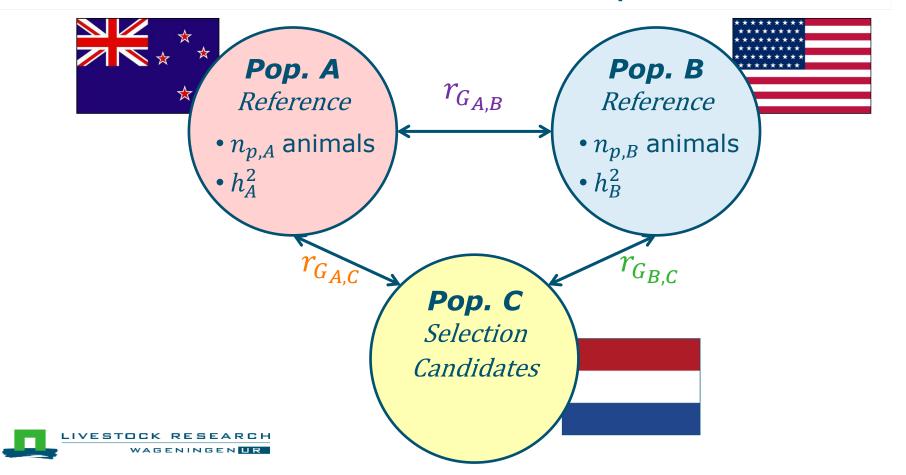
Prediction equation for accuracy of GEBVs using multiple populations in the reference population



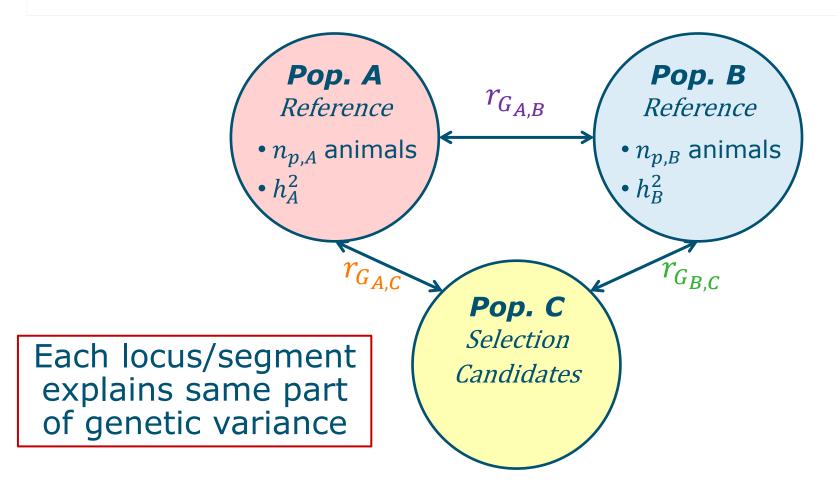
#### Schematic overview



## Schematic overview – an example



#### Schematic overview



### Input parameters

#### Reference population

- Number of animals  $(n_P)$
- Heritability  $(h^2)$
- Genetic correlation  $(r_G)$

## **Selection candidates – Reference population**

- Genetic correlations  $(r_G)$
- Number of independent chromosome segments shared  $(M_e)$



## M<sub>e</sub> across populations

'Effective number of estimated effects'

```
Pop. B \longrightarrow M_e = 4

Pop. C \longrightarrow M_e = 5

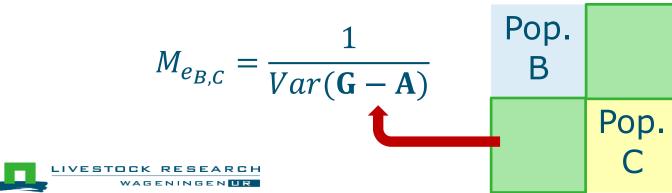
Pop. B - C \longrightarrow M_e = 18
```

## M<sub>e</sub> across populations

'Effective number of estimated effects'

Pop. B 
$$\longmapsto$$
  $M_e = 4$   
Pop. C  $\longmapsto$   $M_e = 5$   
Pop. B - C  $\longmapsto$   $M_e = 18$ 

#### **Relationship matrix**



## Input parameters

#### Reference population

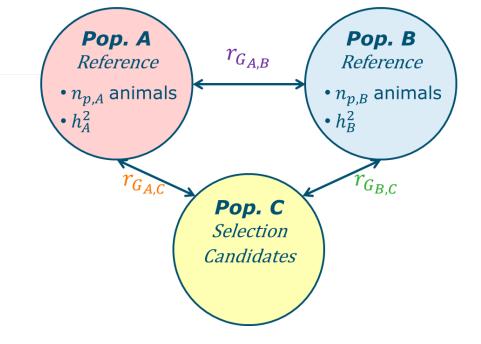
- Number of animals  $(n_P)$
- Heritability  $(h^2)$
- Genetic correlation  $(r_G)$

## **Selection candidates – Reference population**

- Genetic correlations  $(r_G)$
- Number of independent chromosome segments shared  $(M_e)$

$$r_{GEBV_{A+B,C}} = \sqrt{ \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}}}} & r_{G_{B,C}} \sqrt{\frac{h_B^2}{M_{e_{B,C}}}} \end{bmatrix} \begin{bmatrix} \frac{h_A^2}{M_{e_{A,C}}} + \frac{1}{n_{p,A}} & r_{G_{A,B}} \frac{\sqrt{h_A^2 h_B^2}}{\sqrt{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{A,B}} \frac{\sqrt{h_A^2 h_B^2}}{\sqrt{M_{e_{A,C}} M_{e_{B,C}}}} & \frac{h_B^2}{M_{e_{B,C}}} + \frac{1}{n_{p,B}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \end{bmatrix}^{-$$

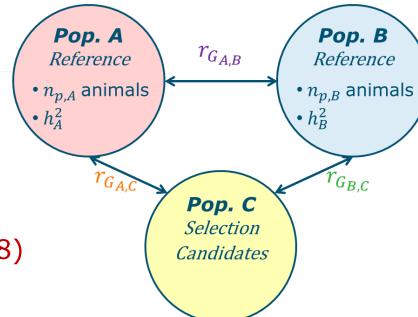
## Derived equation



$$r_{GEBV_{A+B,C}} = \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}}}} & r_{G_{B,C}} \sqrt{\frac{h_B^2}{M_{e_{B,C}}}} \end{bmatrix} \begin{bmatrix} \frac{h_A^2}{M_{e_{A,C}}} + \frac{1}{h_{p,A}} & r_{G_{A,B}} \frac{\sqrt{h_A^2 h_B^2}}{\sqrt{M_{e_{A,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{A,B}} \sqrt{\frac{h_A^2 h_B^2}{M_{e_{A,C}} M_{e_{B,C}}}} & \frac{h_B^2}{M_{e_{B,C}}} + \frac{1}{h_{p,B}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_B^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}}} \\ r_{G_{B,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{B,C}}}}} \\ r_{G_{A,C}} \sqrt{\frac{h_A^2}{M_{e_{A,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}}$$

## Derived equation

- 1 population in reference
  - → Equal to Daetwyler et al. (2008)



$$r_{GEBV_{A+B,C}} = \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{(h_A^2)}{M_{e_{A,C}}}} & r_{G_{B,C}} \sqrt{\frac{(h_B^2)}{M_{e_{B,C}}}} \end{bmatrix} \begin{bmatrix} \frac{(h_A^2)}{M_{e_{A,C}}} + \frac{1}{(n_{p,A})} & r_{G_{A,B}} \frac{\sqrt{(h_A^2 h_B^2)}}{\sqrt{M_{e_{A,C}} M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{(h_A^2)}{M_{e_{A,C}}}} \\ r_{G_{A,C}} \sqrt{\frac{(h_A^2)}{M_{e_{A,C}}}} & \frac{(h_A^2)}{M_{e_{B,C}}} \end{bmatrix}^{-1} \\ r_{G_{B,C}} \sqrt{\frac{(h_A^2)}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \begin{bmatrix} r_{G_{A,C}} \sqrt{\frac{(h_A^2)}{M_{e_{A,C}}}} \\ r_{G_{B,C}} \sqrt{\frac{(h_A^2)}{M_{e_{B,C}}}} \end{bmatrix}^{-1} \end{bmatrix}$$

#### Simulations for validation



## Genotypes of 1033 HF animals

422 405 SNPs 50 000 candidate QTL (Random or Low MAF)



#### **Phenotypes**

Simulated (4000 QTL)

TBV + E

#### Simulations for validation



## **Genotypes of 1033 HF animals**

422 405 SNPs 50 000 candidate QTL (Random or Low MAF)



#### **Phenotypes**

Simulated (4000 QTL) TBV + E



## **Breeding value (GEBV) estimation**

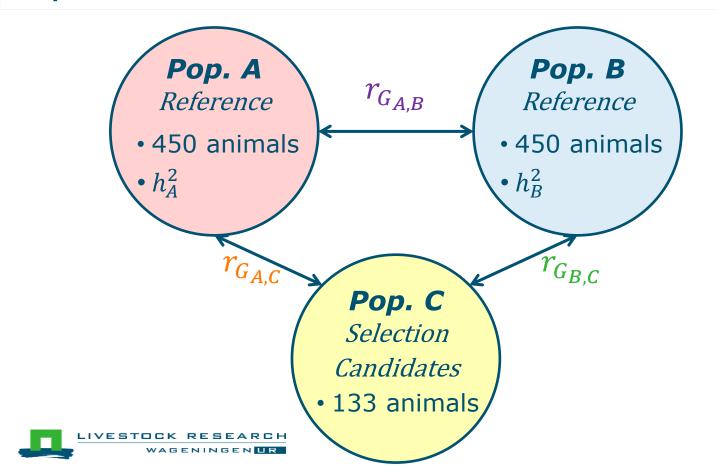
Multi-trait GBLUP



#### **Empirical Accuracy**

Correlation GEBV with TBV

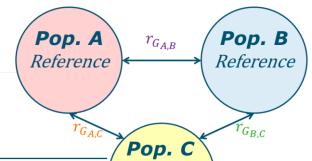
## Populations for validation



#### **Division:**

Half-sib families kept in same population

### Scenarios for validation

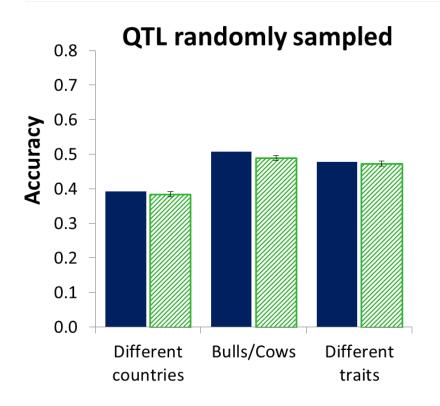


Selection Candidates

	Heritability (h²)		Genetic correlation $(r_G)$		
Scenarios	Pop. A	Pop. B	Pop. A - B	Pop. A – C	Pop. B - C
Different countries	0.95	0.95	0.6	0.8	0.4
Bulls/cows	0.95	0.30	1.0	1.0	1.0
Different traits	0.95	0.30	0.6	1.0	0.6

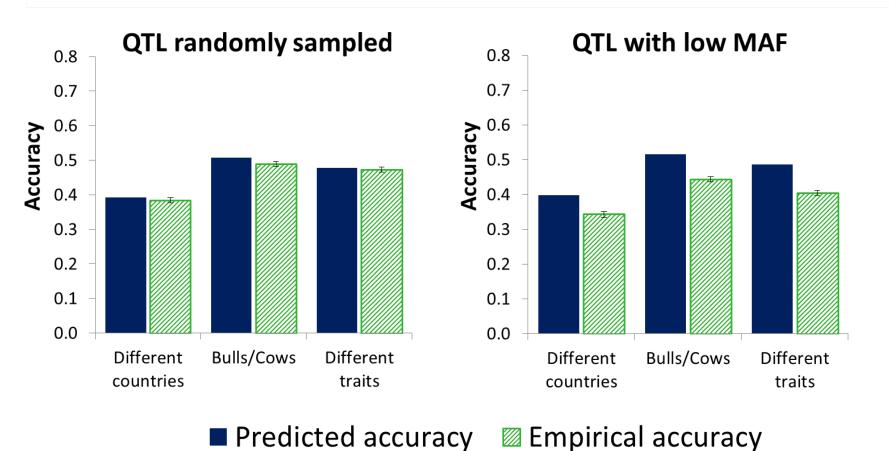


#### Results of validation

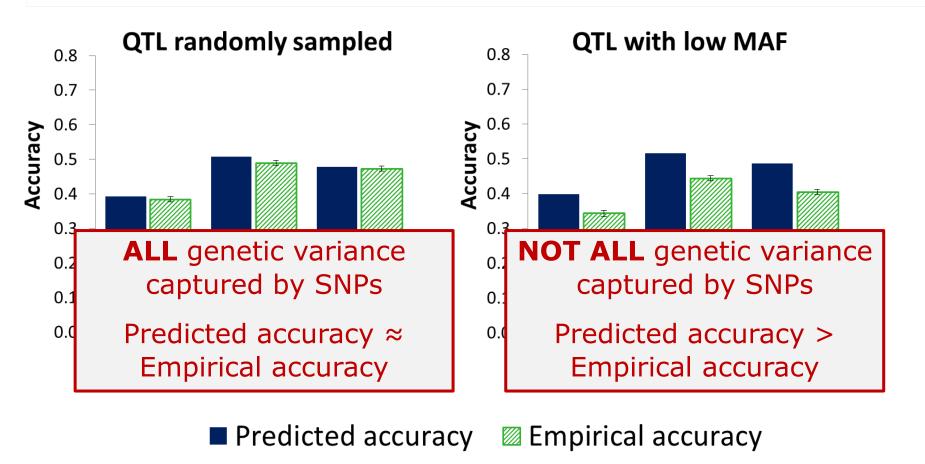


Predicted accuracy
Empirical accuracy

#### Results of validation



#### Results of validation



#### Conclusion

# **Equation accurately predicts GEBV accuracy**

- Multiple populations:
  - Countries
  - Bulls/Cows
  - Traits
  - Breeds?
- All genetic variance captured





#### Conclusion

# **Equation accurately predicts GEBV accuracy**

- Multiple populations:
  - Countries
  - Bulls/Cows
  - Traits
  - Breeds?
- All genetic variance captured



yvonne.wientjes@wur.nl

