A 2-step dynamic linear model for milk yield forecasting and mastitis detection

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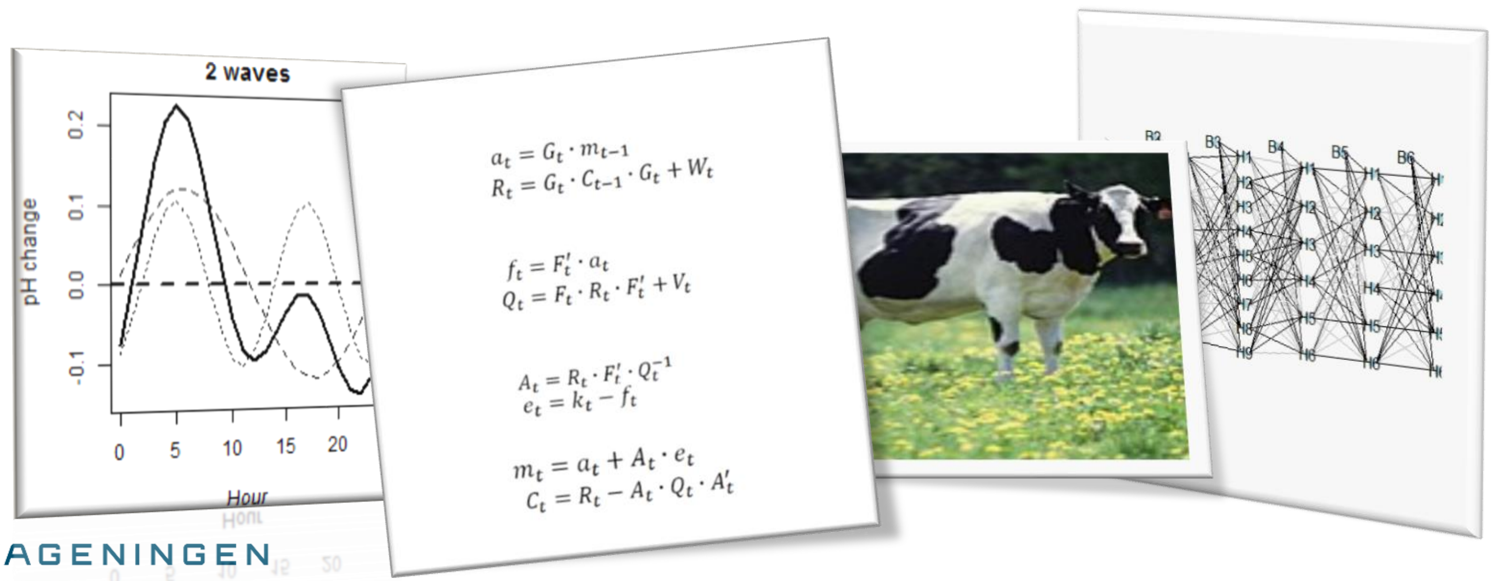

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Forecast Milk Yield per Milking Session

Aims of the study:

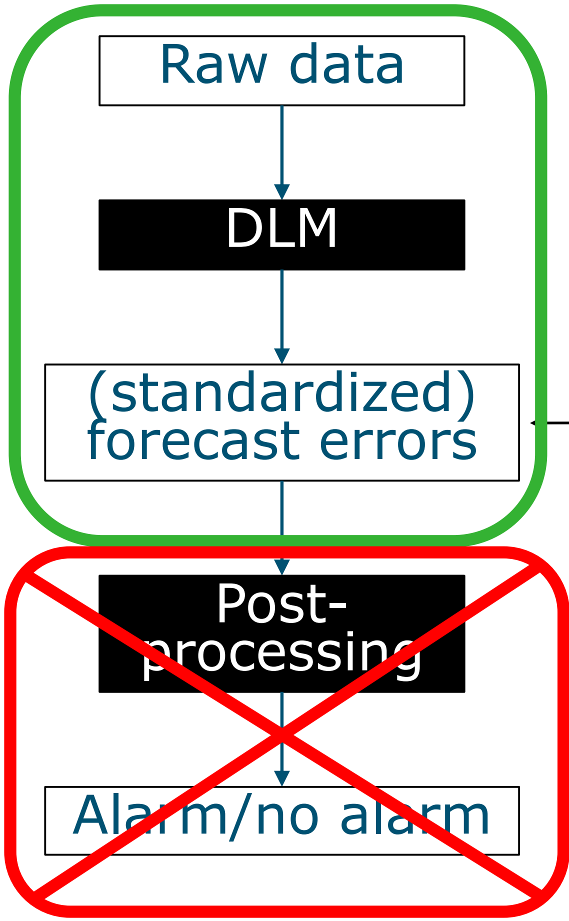
1. To implement and **demonstrate DLM** for forecasting milk yield per milking session
2. To test the **importance of the farm-specific** implementation of the DLM
3. To test the **effect of SCC** on the forecast accuracy of the DLM.

What the DLM tells us for each milking session (the two steps):

- How much milk do we expect **in total for today**?
- **What percentage** of that total milk yield do we expect to see at this session?

The DLM in general

What’s the point of a DLM?



The normality hypothesis:

IF “everything is fine” ***THEN*** “things progress as expected”

Therefore:

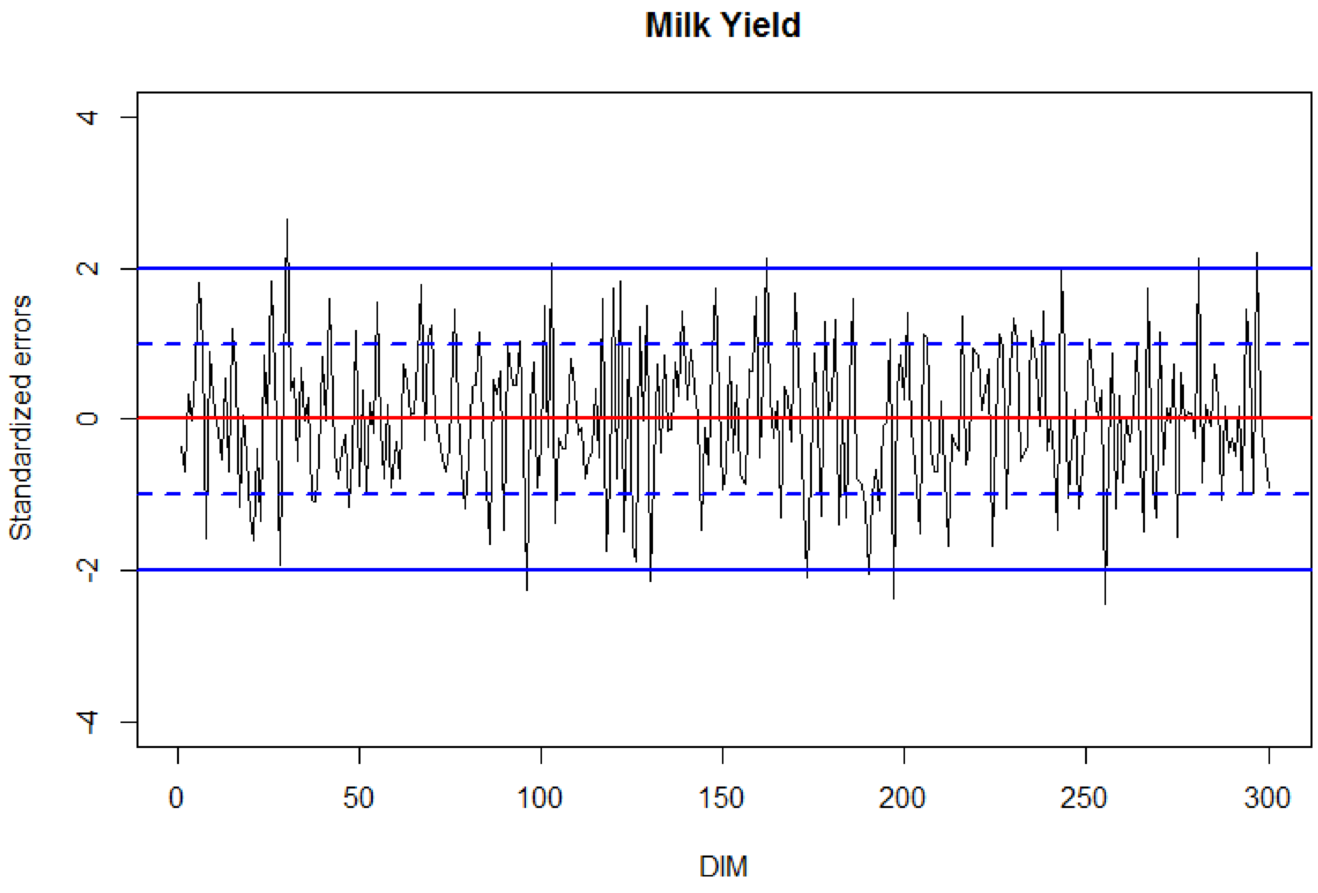
IF “things progress UN-expectedly” ***THEN*** “Something is wrong!”

$$u_t = \frac{e_t}{\sqrt{Q_t}}$$

$$= \frac{Observed_t - Forecasted_t}{\sqrt{Forecast\ variance}}$$

What's the point of a DLM?

- Standardized forecast errors - mutually independent values!



Dynamic Linear Models

How?

Structure:

- ***Observation equation***

$$\boldsymbol{Y}_t = \boldsymbol{F}_t'\boldsymbol{\theta}_t + \boldsymbol{v}_t, \qquad \boldsymbol{v}_t \sim N\left(\underline{0}, \boldsymbol{V}\right)$$

- ***System equation***

$$\boldsymbol{\theta}_t = \boldsymbol{G}_t\boldsymbol{\theta}_{t-1} + \boldsymbol{w}_t, \qquad \boldsymbol{w}_t \sim N\left(\underline{0}, \boldsymbol{W}\right)$$

θ_t	*Parameter vector*
F_t	*Design matrix*
G_t	*System matrix*
V	*Observational variance*
W	*System variance*

Dynamic Linear Models

How? – does it adapt to the individual cow

- Prior information:

$$a_t = G_t \cdot m_{t-1}$$ Prior mean

$$R_t = G_t \cdot C_{t-1} \cdot G_t + W_t$$ Prior variance

- 1-step forecast information:

$$f_t = F_t' \cdot a_t$$ Forecast

$$Q_t = F_t \cdot R_t \cdot F_t' + V_t$$ Forecast variance

- Adaptation information

$$A_t = R_t \cdot F_t' \cdot Q_t^{-1}$$ Adaptive coefficient

$$e_t = k_t - f_t$$ Forecast error

- Filtered (posterior) information

$$m_t = a_t + A_t \cdot e_t$$ Filtered mean

$$C_t = R_t - A_t \cdot Q_t \cdot A_t'$$ Filtered variance

8 lines of math

The DLM
in this study

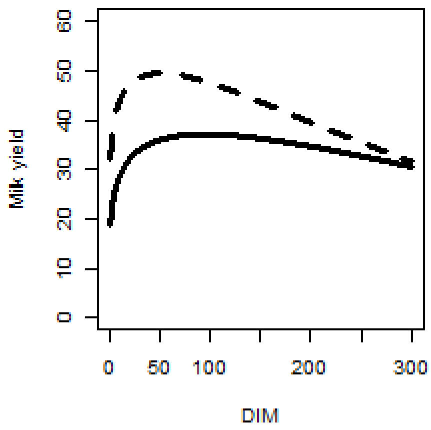
Forecast Milk Yield per Milking

Strategy

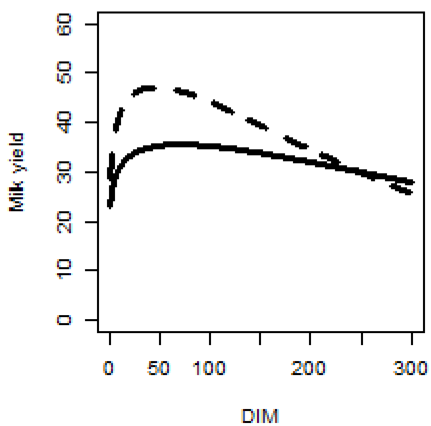
θ_t	*Parameter vector*
F_t	*Design matrix*
G_t	*System matrix*

"How much milk do we expect **in total for today**?"

Farm: 4672.DE



Farm: 8773.FR



$$\boldsymbol{Y}_t = \boldsymbol{F}_t' \boldsymbol{\theta}_t + \boldsymbol{v}_t, \qquad \boldsymbol{v}_t \sim N\left(\underline{0}, \boldsymbol{V}\right)$$

$$\boldsymbol{\theta}_t = \boldsymbol{G}_t \boldsymbol{\theta}_{t-1} + \boldsymbol{w}_t, \qquad \boldsymbol{w}_t \sim N\left(\underline{0}, \boldsymbol{W}\right)$$

$$\theta_{t+1} = \begin{bmatrix} \widehat{MY}(DIM_t)_t + d\widehat{MY}(DIM_t)_t \cdot T \\ T \end{bmatrix}$$

$$d\widehat{MY}(DIM_t)_t = a \cdot DIM_t^{\,b} \cdot e^{-c \cdot DIM} - a \cdot DIM_{t-1}^{\,b} \cdot e^{-c \cdot DIM_{t-1}}$$

$$DIM_t = DIM_{t-1}$$

$$\rightarrow dMY_t^{DIM} = 0$$

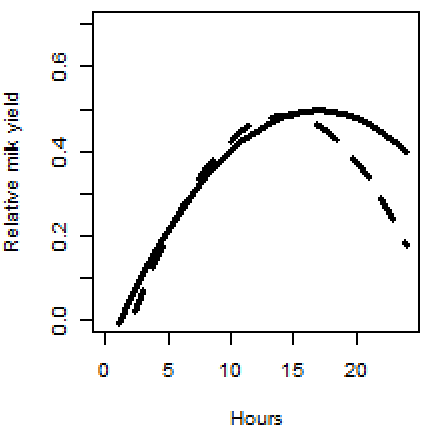
Forecast Milk Yield per Milking

θ_t	*Parameter vector*
F_t	*Design matrix*
G_t	*System matrix*

Strategy

"**What percentage** of that total milk yield do we expect to see at this milking session?"

Farm: 4672.DE

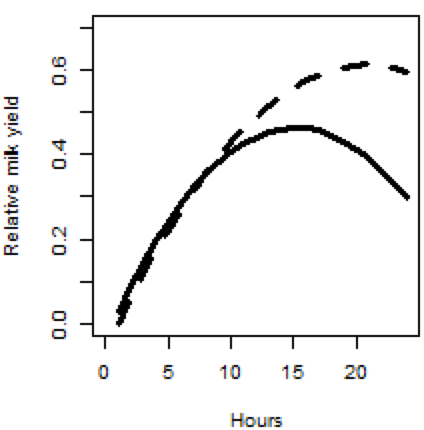


$$\boldsymbol{Y_t = F_t'\theta_t + v_t}, \qquad \boldsymbol{v_t} \sim N\left(\underline{0}, \boldsymbol{V}\right)$$

$$\boldsymbol{\theta_t = G_t\theta_{t-1} + w_t}, \qquad \boldsymbol{w_t} \sim N\left(\underline{0}, \boldsymbol{W}\right)$$

$$Y_t = \widehat{MY}(DIM_t)_t \cdot \widehat{MY}_{relative}(Interval_t)_t$$

Farm: 8773.FR

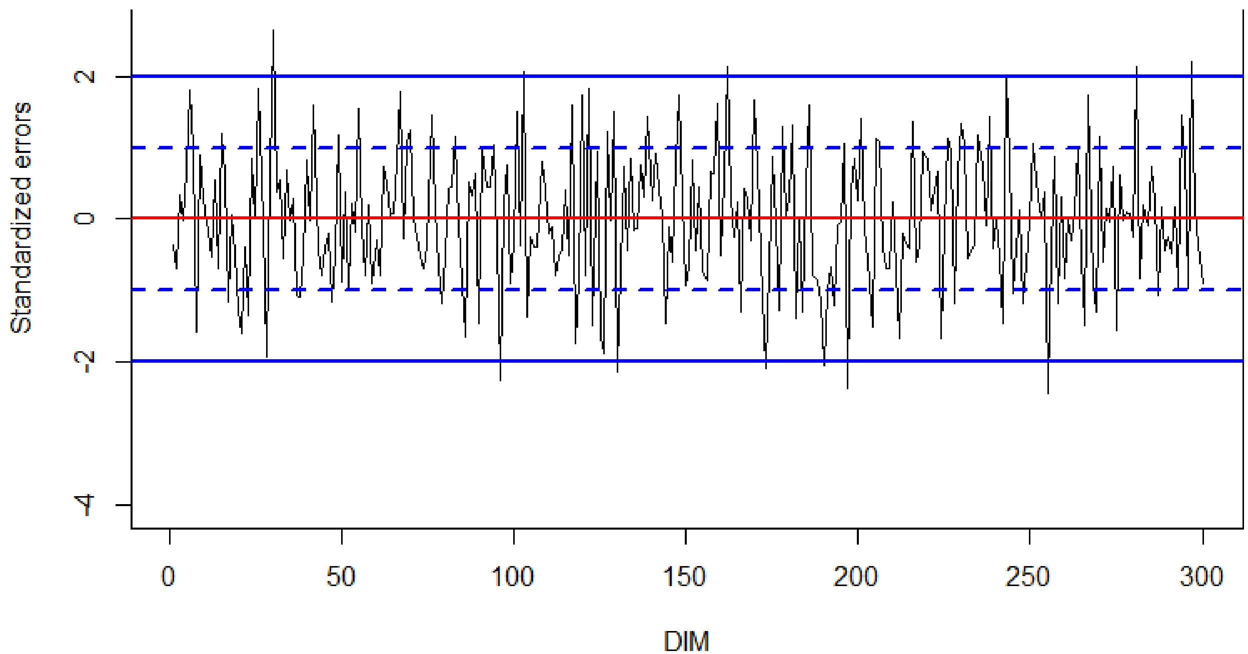


$$\widehat{MY}_{relative}(Interval_t)_t = \alpha_0 + \alpha_1 \cdot Interval_t + \alpha_2 \cdot Interval_t^2$$

Testing the effects

Testing the effects

- Standardized forecast errors - mutually independent!
- But how are they affected by:
    - Lactation stage (early, middle, late)?
    - SCC level (elevated, not elevated)?
    - Modelling strategy (“proper”, “random”)?



Testing the effects

Mixed effects model

R function: lme

Dependent variable:	Standardized forecast error
Independent variables:	SCC level (2 levels)
	Lactation stage (3 levels)
	Modeling strategy (2 levels)
	+ all 2-way interactions
	+ the 3-way interaction
Random effects:	Farm/Cow

Results
&
Conclusions

Forecast Milk Yield per Milking Session

Results

Table 4. The results of the ANOVA test applied to the mixed effects model describing the standardized forecast errors

Variable	df	P-value
(Intercept)	1	0.64
Lactation stage	2	0.68
Modeling strategy	1	0.77
SCC level	1	<0.0001
Lactation stage: Modeling strategy	2	0.95
Lactation stage: SCC level	2	<0.0001
Modeling strategy: SCC level	1	0.46
Lactation stage: Modeling strategy: SCC level	2	0.94

Conclusions

DLM for milk yield forecast per milking session

- How much milk do we expect **in total for today**?
- **What percentage** of that total milk yield do we expect to see at this milking session?
- **Dynamically adapt** to the individual cow

Importance of the farm-specific implementation

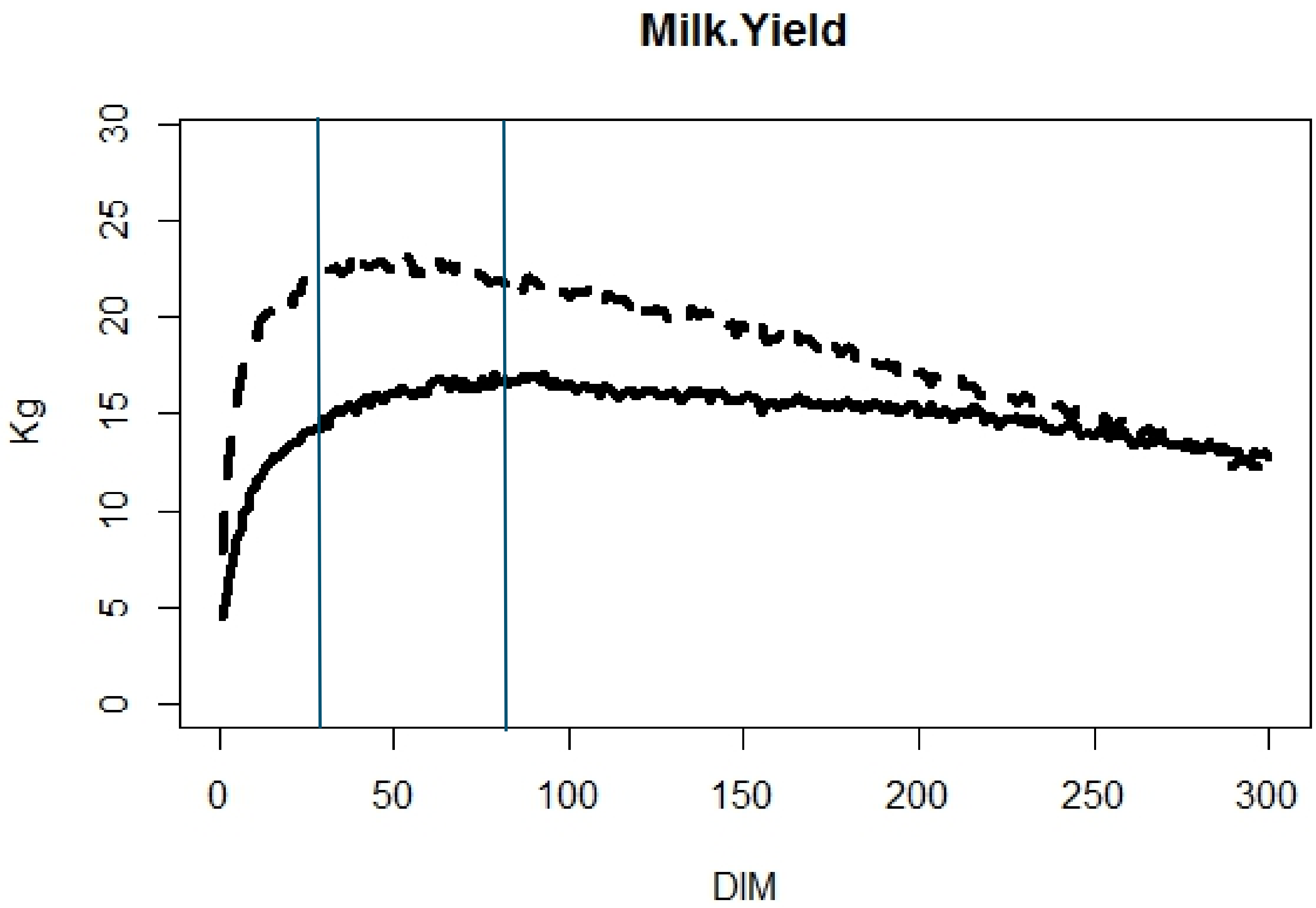
- **No significant effect** of modelling strategy was found

The effect of SCC on the forecast accuracy

- **A significant effect** was seen
  – suggests potential for use in mastitis alarm system!

Extra

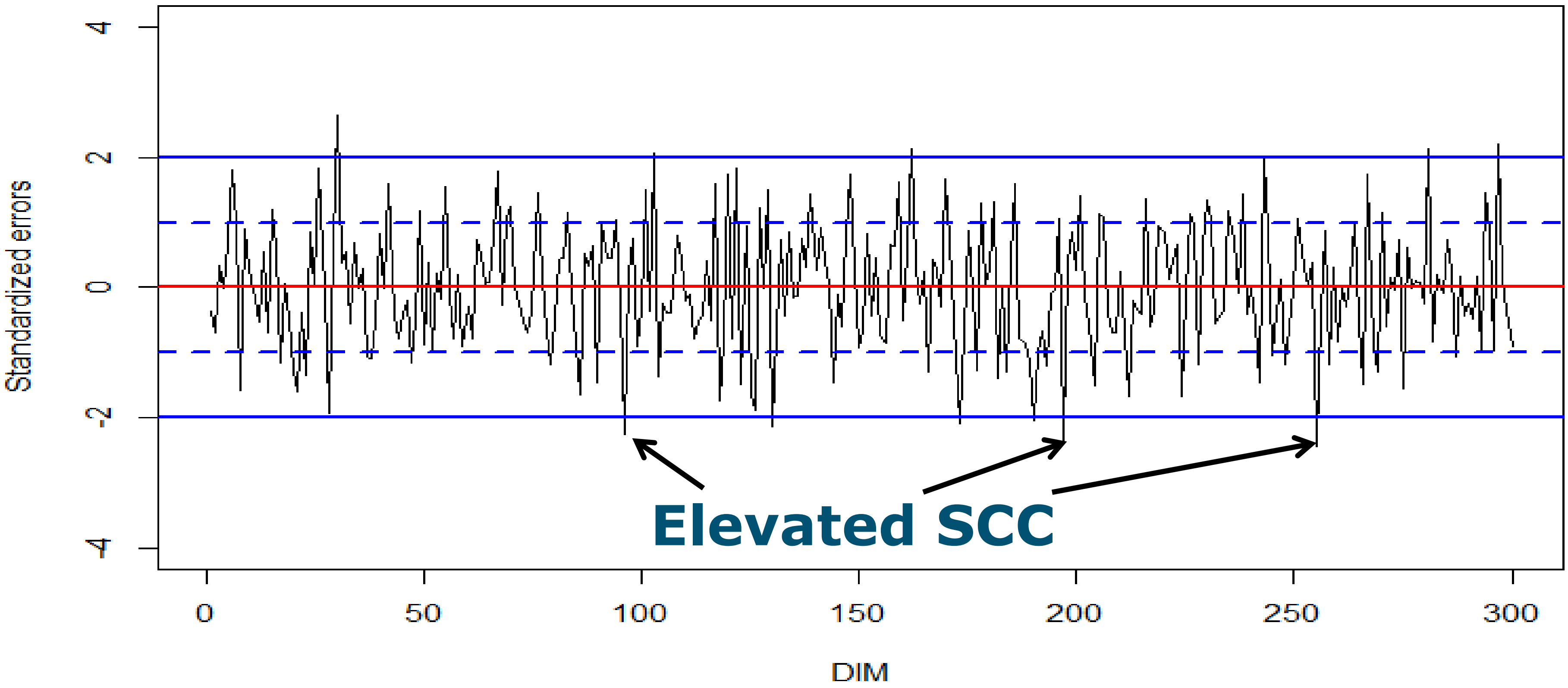
Early, middle, late – explained!



Testing the effects

Elevated Somatic Cell Count

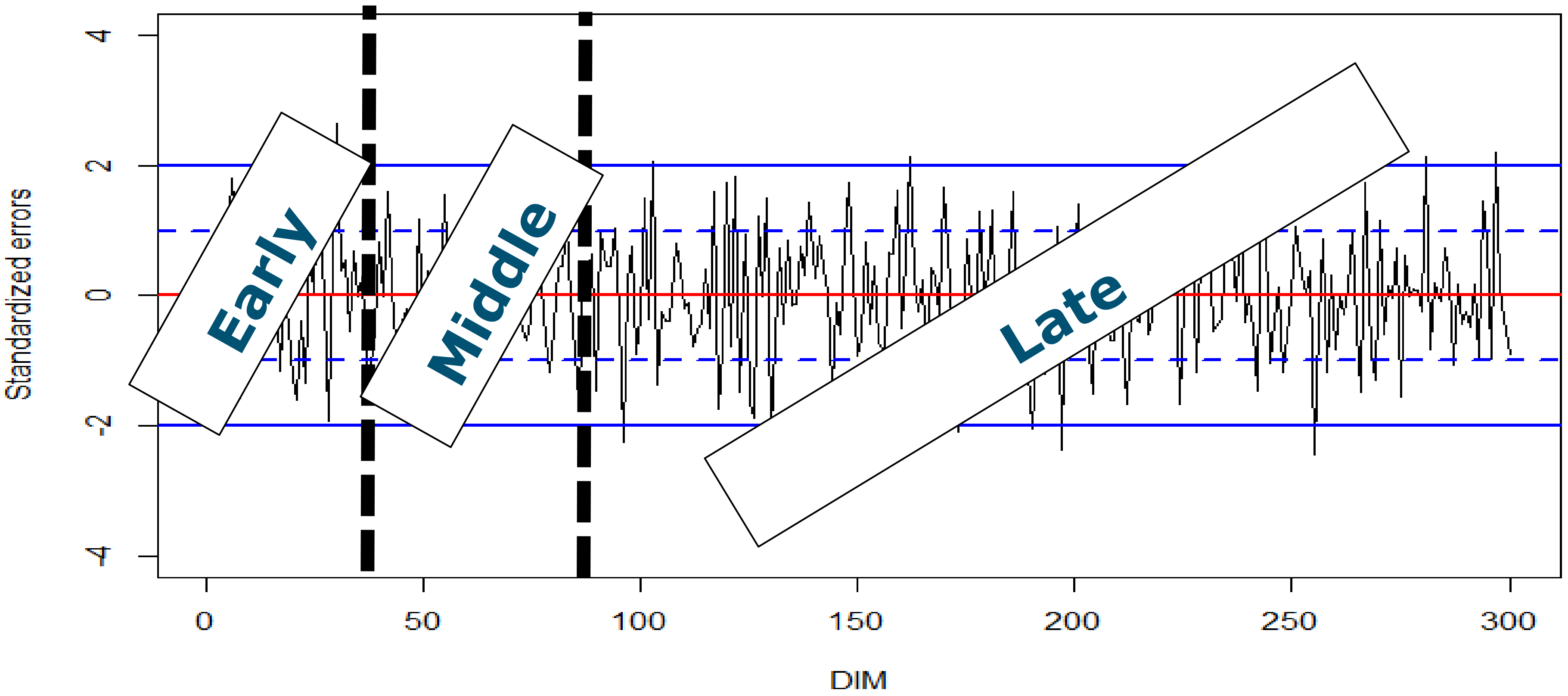
- Primiparious: > 150.000 cells/ml
- Multiparious: > 250.000 cells/ml



Testing the effects

Lactation stage

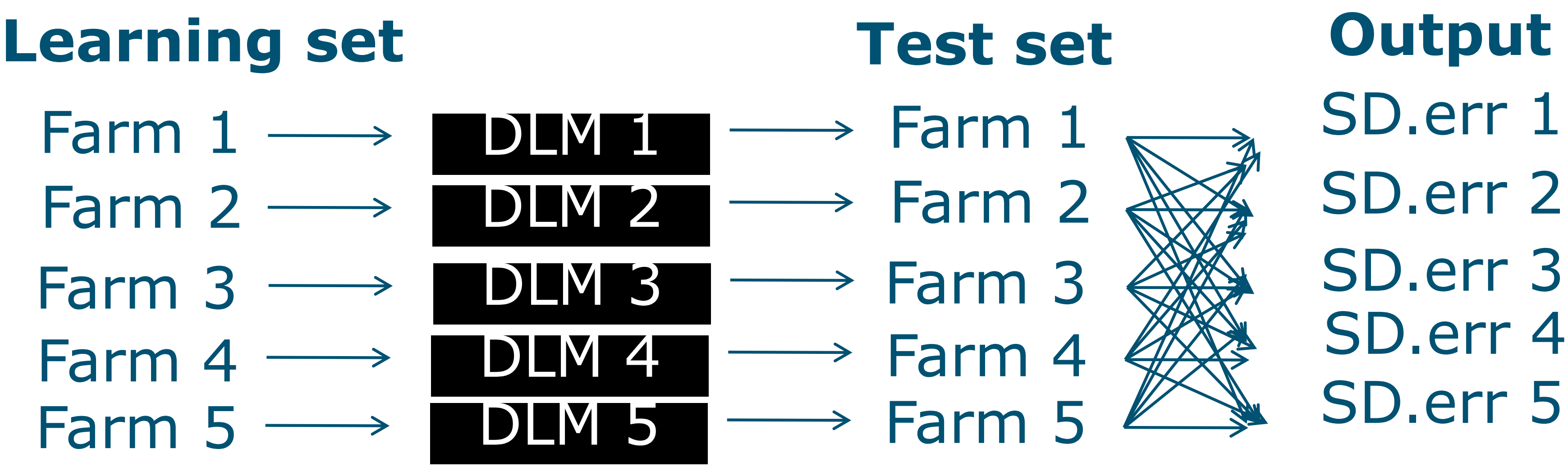
- Standardized forecast errors - mutually independent values!



Testing the effects

Modelling strategies

"Random"



Thus, the milk yield of all cows were modeled with
1 "proper" model
and 4 "random" models